Math 20550 - Calculus III - Summer 2014
June 20, 2014
Exam 2

Name: ____

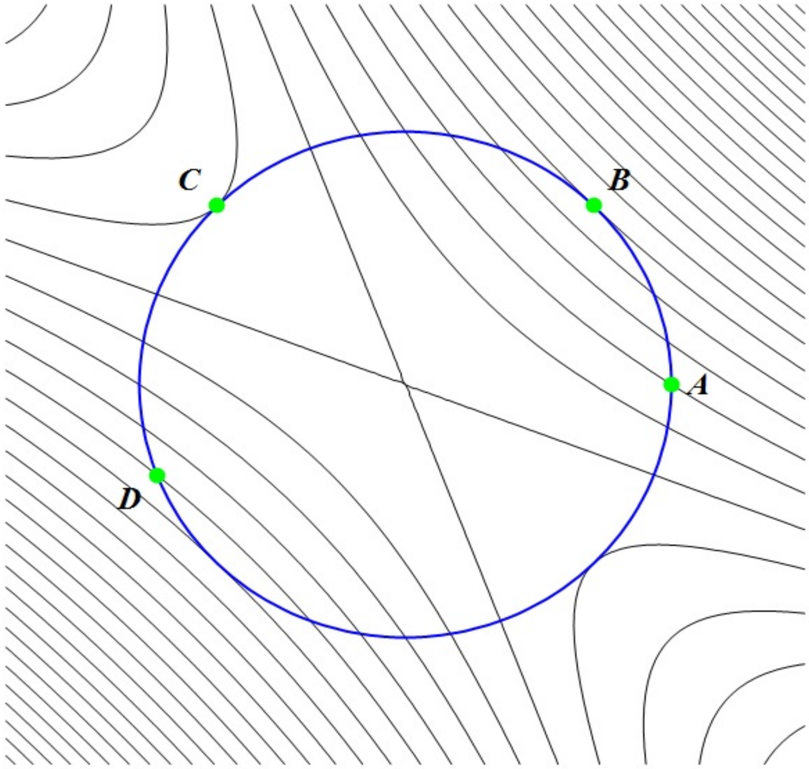
There is no need to use calculators on this exam. This exam consists of 12 problems on 12 pages. You have 75 minutes to work on the exam. There are a total of 105 available points and a perfect score on the exam is 100 points. All electronic devices should be turned off and put away. The only things you are allowed to have are: a writing utensil(s) (pencil preferred), an eraser, your 3×5 notecard, and an exam. No notes (other than the aforementioned notecard), books, or any other kind of aid are allowed. All answers should be given as exact, closed form numbers as opposed to decimal approximations (i.e., π as opposed to 3.14159265358979...). **You must show all of your work to receive credit. Please box your final answers.** Cheating is strictly forbidden. Good luck!

Honor Pledge: As a member of the Notre Dame community, I will not participate in, nor tolorate academic dishonesty. My signature here binds me to the Notre Dame Honor Code:

Signature:____

Problem	Score
1	/5
2	/15
3	/10
4	/5
5	/10
6	/5
7	/10
8	/10
9	/10
10	/10
11	/5
12	/10
Score	/100

Problem 1 (5 points)**.** *Consider the following contour plot for a function $f(x, y)$:*



The circle is a level curve $g(x, y) = k$. Which of the following must ALWAYS be true?

I. Subject to $g(x, y) = k$, $f(x, y)$ has a possible extremum at C.

II. Subject to $g(x, y) = k$, $f(x, y)$ has a possible maximum at A.

III. Subject to $g(x, y) = k$, $f(x, y)$ has a possible minimum at D.

IV. Subject to $g(x, y) = k$, $f(x, y)$ has an absolute maximum at B.

V. $f(x, y)$ has an absolute maximum or absolute minimum at C.

Answer:____________

Problem 2 (15 points - 5 points each)**.** *Consider the curve parametrized by*

$$\mathbf{r}(t) = \left\langle 2t, t^2, \frac{1}{3}t^3 \right\rangle, \quad 0 \le t \le 2.$$

(a) *Find the arc length of the curve.*
(b) *Find the curvature at the point* $\left(2, 1, \frac{1}{3}\right)$.
(c) *Find an equation for the osculating plane at the point* $\left(2, 1, \frac{1}{3}\right)$.

Problem 3 (10 points)**.** *Let* $z = f(x, y)$*, where* f *is a differentiable function and suppose that* $x = x(s, t)$ *and* $y = y(s, t)$*. Given that*

$$x(1,0) = 2, \quad x_s(1,0) = -2, \quad x_t(1,0) = 6, \quad f_x(2,3) = -1$$

$$y(1,0) = 3, \quad y_s(1,0) = 5, \quad y_t(1,0) = 4, \quad f_y(2,3) = 10$$

Find $\dfrac{\partial z}{\partial t}(1,0)$.

Problem 4 (5 points)**.** *Compute* $\int \mathbf{r}(t)dt$ *where*

$$\mathbf{r}(t) = t\mathbf{j} + e^t\mathbf{k}.$$

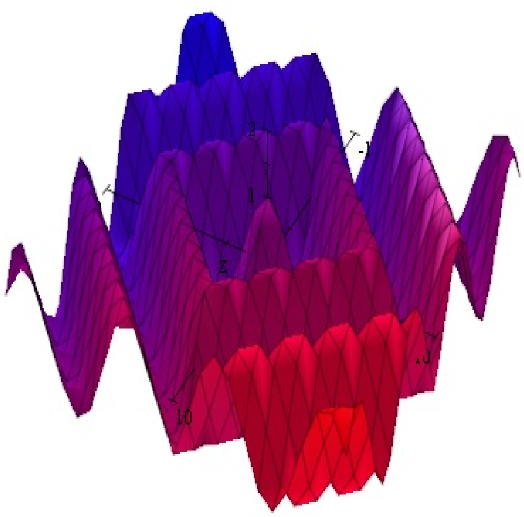
Problem 5 (10 points). *Suppose z is defined implicitly by*

$$z = e^x \sin(y + z).$$

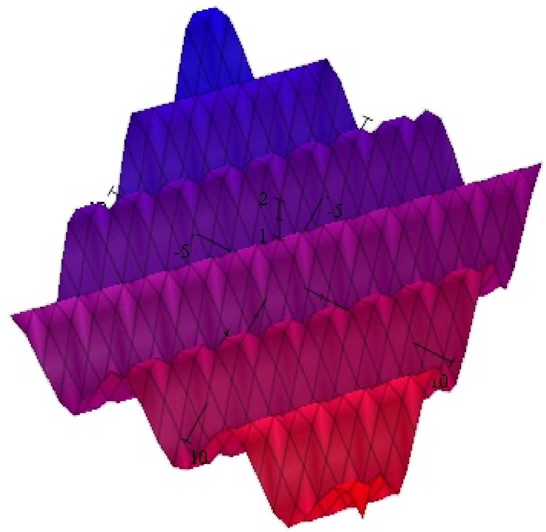
Find all first partials of z.

Problem 6 (5 points)**.** *Which of the following is the graph of*

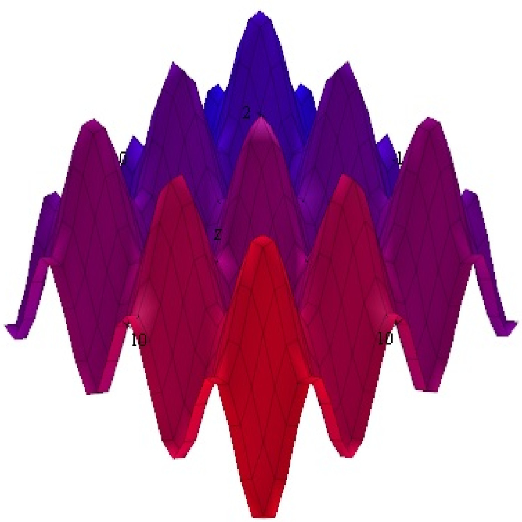
$$f(x, y) = \cos(|x| + |y|)?$$



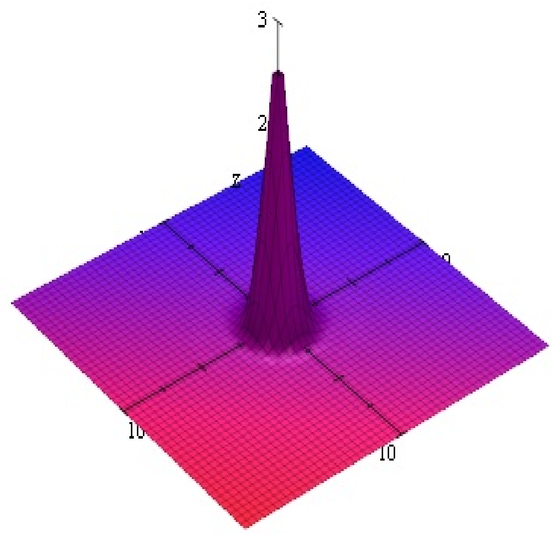
I.



II.



III.



IV.

*Ans:*____________

Problem 7 (10 points - 5 points each). *Consider the function* $f(x, y) = x \tan y$.

(a) *Compute the derivative of* f *at* $\left(2, \frac{\pi}{4}\right)$ *in the direction of* $\mathbf{v} = \mathbf{i} + \mathbf{j}$.

(b) *In which direction from* $\left(2, \frac{\pi}{4}\right)$ *is* f *decreasing the fastest?*

Problem 8 (10 points). *Find three positive numbers whose sum is* 15 *and the sum of whose squares is as small as possible.*

Problem 9 (10 points). *Find the tangential and normal components of acceleration for a particle traveling along the trajectory*

$$\mathbf{r}(t) = (3t - t^3)\mathbf{i} + 3t^2\mathbf{j}.$$

Problem 10 (10 points)**.** *Find an equation for the tangent plane to* $x^4 + y^4 + z^4 = 3x^2y^2z^2$ *at the point* $(1, 1, 1)$.

Problem 11 (5 points)**.** *Sketch the curve represented by*

$$\mathbf{r}(\theta) = \langle 3\sec\theta, 2\tan\theta\rangle \,.$$

Make sure to indicate the orientation of the curve (direction of increasing θ).

Problem 12 (10 points)**.** *A particle with mass 2 starts at the origin with an initial velocity* $\mathbf{v}_0 = \mathbf{i} - \mathbf{j} + 3\mathbf{k}$. *Its acceleration is* $\mathbf{a}(t) = 6t\mathbf{i} + 12t^2\mathbf{j} - 6t\mathbf{k}$. *Find its position and momentum functions. (Recall that its momentum,* $\mathbf{p}(t)$*, is mass times velocity.)*